PROPAGATION OF ELECTROMAGNETIC DISTURBANCES AND STABILITY OF STATIONARY STATES IN MEDIA WITH A NONLINEAR OHM'S LAW

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The principles of propagation and development of small disturbances in nonlinear conducting media are studied.

It is well known that in some media, for example, in nonequilibrium plasma [1] and semiconductors [2], there is a nonlinear relationship between current density **j** and electric field **E**. This relationship is usually determined from examination of charge carrier kinetics, or from experiment.

One of the possible approaches for the description of electrical phenomena in nonlinear conductors is the direct introduction of a model function **j**(**E**) in the system of electrodynamic equations. Such a function treats the material equation of the medium as given and completes the system of Maxwell equations. This approach was used in several studies [3-8] and will evidently be developed further.

The introduction of a nonlinear ohmic law adds additional difficulties to the procedure of solving stationary-state problems, connected not only with the nonlinearity of the original equations, but also with the change in type of these equations [3] in areas with a falling volt–ampere characteristic $j = j(E)$. The latter situation may lead to instability of the stationary state [4].

The instability of homogeneous stationary current distributions with negative differential conductivity ($\sigma_d = dj/dE < 0$) has been examined repeatedly in the literature (see, e.g., [2]). In these studies the analysis of small disturbances was conducted on the basis of the system of field equations and various partial solutions of the kinetic equations for charge carriers. Due to the unwieldiness of the system, only potential disturbances of the electric field have been strictly examined, as a rule. The use of a model characteristic **j**(**E**) permits a more detailed study of propagation of all possible electromagnetic disturbances and the initial stage of development of instability, independent of the mechanism of nonlinear conductivity.

This study will examine the effect of anisotropy in propagation of small disturbances, and the various types of waves in nonlinearly conductive media. The criteria for time damping of arbitrarily oriented harmonics of initial fluctuations occurring on a background of homogeneous current distribution in an infinite space are found. The damping criteria for amplitude of harmonic oscillations oriented in the direction of wave propagation are also found.

A formulation of the stability problem for a bounded region is given. For a homogeneous current state the example of incremental turbulent disturbances for the case $\sigma_d < 0$ is constructed. The asymptotic stability of inhomogeneous current distributions for $\sigma > 0$ is proven by the energy method. The question of a connection between the stability conditions and the extremal principle of minimum Joulean dissipation is examined. It is shown that for a nonlinearly conductive medium this principle is inadequate as a stability condition even for the relatively simple case examined in [9].

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1. An Analysis of the Dispersion Equations and Various Types of Disturbances

We will examine immobile conductors with values of dielectric and magnetic permeability close to unity. The system of electromagnetic equations in this case will have the form

$$\operatorname{rot}\mathbf{E} = -c^{-1}\partial\mathbf{H}/\partial t, \quad \operatorname{rot}\mathbf{H} = 4\pi c^{-1}\mathbf{j} + c^{-1}\partial\mathbf{E}/\partial t \tag{1.1}$$

$$\operatorname{div}\mathbf{E} = 4\pi\rho_e, \quad \operatorname{div}\mathbf{H} = 0, \ \mathbf{j} = \sigma(E)\,\mathbf{E}$$

The description of Ohm's law in the last equation of Eq. (1.1) assumes a single-valued function $\mathbf{j}(\mathbf{E})$, which corresponds to a monotonic or N-shaped volt–ampere characteristic. We will examine disturbances of a homogeneous state which is characterized by a given point (E_0, j_0) on this curve. In the case where the characteristic is S-shaped, i.e., the function $j(E)$ is not single-valued, in analysis of small disturbances only the one single-valued branch containing the point (E_0, j_0) must be examined. It need only be assumed that the current studied does not unite the two single-valued branches.

The use of Ohm's law in the form of Eq. (1.1) in describing nonstationary processes is based on a series of simplified assumptions on the dynamics of the electron gas. In particular, the following conditions must be satisfied:

$$\omega \ll \nu_e, \qquad \omega_e \ll \nu_e \tag{1.2}$$

Here, ω is the characteristic frequency of the process, ν_e is the effective electron collision frequency, and ω_e is the Larmor frequency. If the first inequality of Eq. (1.2) is not fulfilled, electron inertia must be considered, while if the second is not fulfilled, the Hall effect must be considered. The Maxwell equations contain a displacement current which becomes apparent for $\omega \gtrsim 4\pi\sigma$. Frequency ranges for which the latter inequality is compatible with the condition $\omega \ll \nu_e$ can exist for quite low conductivity, for example, in the case of a low degree of ionization.

We will examine a homogeneous stationary state of the medium

$$\mathbf{E} = \mathbf{E}_0 = \text{const}, \qquad \mathbf{j} = \mathbf{j}_0 = \sigma(E_0)\,\mathbf{E}_0 = \text{const} \tag{1.3}$$

Equation (1.3) is characterized by an inhomogeneous magnetic field of the form

$$\mathbf{H}_0 = 2\pi c^{-1}(\mathbf{j}_0 \times \mathbf{x}) + \nabla\Phi$$

where Φ is an arbitrary harmonic function. In the case of infinite current space the field $\mathbf{H}_0$ is unlimited for $|\mathbf{x}| \to \infty$ independent of the choice of a potential Φ.

At this point disturbances throughout the entire space will be examined. The results of the analysis conducted below will qualitatively depict the behavior of disturbances in a finite region far from borders, provided the linear dimensions of the region significantly exceed the scale of the fluctuations.

Small disturbances applied to an arbitrary inhomogeneous stationary state will satisfy the following linear system:

$$\begin{gathered}\operatorname{rot}\delta\mathbf{E} = -c^{-1}\partial\delta\mathbf{H}/\partial t, \qquad \operatorname{rot}\delta\mathbf{H} = 4\pi c^{-1}\delta\mathbf{j} + c^{-1}\partial\delta\mathbf{E}/\partial t \\ \delta\mathbf{j} = \sigma_d(\mathbf{l}\delta\mathbf{E})\,\mathbf{l} + \sigma(\delta\mathbf{E} - (\mathbf{l}\delta\mathbf{E})\,\mathbf{l}) \equiv \sigma_d\delta\mathbf{E}_\parallel + \sigma\delta\mathbf{E}_\perp\end{gathered} \tag{1.4}$$

Here, $\mathbf{l}$ is a unit vector directed along the undisturbed electric field $\mathbf{E}_0 = E_0\mathbf{l}$, and the vectors $\delta\mathbf{E}_\parallel$ and $\delta\mathbf{E}_\perp$ are defined as the components parallel to and perpendicular to $\mathbf{E}_0$ of the disturbance $\delta\mathbf{E}$. The regular conductivity σ and differential conductivity $\sigma_d = dj/dE$ in Eqs. (1.4) are taken in the undisturbed state. In the case of a homogeneous state [Eq. (1.3)] these values will be constant. System (1.4) does not include the equation $\operatorname{div}\delta H = 0$, which plays the role of an initial condition, nor the equation $\operatorname{div}\delta E = 4\pi\delta\rho_e$, which serves to determine the fluctuation in charge density. As a consequence of the nonlinear ohmic law, the relationship between small disturbances $\delta\mathbf{j}$ and $\delta\mathbf{E}$ proves to be of a tensor nature. The last equation of (1.4) can be written in the form $\delta\mathbf{j} = \boldsymbol{\sigma}\cdot\delta\mathbf{E}$, where $\boldsymbol{\sigma}$ is a symmetric tensor of second rank. For $\sigma_d \neq \sigma$, "Ohm's law" for disturbances is characterized by spherical anisotropic conductivity.

As a consequence of the symmetry of the tensor $\boldsymbol{\sigma}$ there always exist disturbances $\delta\mathbf{E}$, for which $\delta\mathbf{j} \parallel \delta\mathbf{E}$ (in contrast to the case when conductivity anisotropy is produced by the Hall effect). Any Cartesian base $\mathbf{e}_i$ such that $\mathbf{e}_1 = \mathbf{l}$, forms a system of main axes for the tensor $\boldsymbol{\sigma}$. In this base, only the diagonal elements of the matrix $\|\sigma_{ij}\|$ are nonzero, while

$$\sigma_{11} = \sigma_d, \qquad \sigma_{22} = \sigma_{33} = \sigma$$

For the case of a homogeneous stationary state [Eq. (1.3)] we will examine an arbitrarily oriented one-dimensional disturbance of the type

$$(\delta\mathbf{E},\ \delta\mathbf{j},\ \delta\mathbf{H}) = \mathrm{Re}\,\{(\mathbf{E}',\ \mathbf{j}',\ \mathbf{H}')\, e^{i(\mathbf{kx}-\omega t)}\} \tag{1.5}$$

In Eq. (1.5) the complex amplitudes $\mathbf{E}'$, $\mathbf{j}'$, $\mathbf{H}'$ are constant. The wave vector $\mathbf{k}$ and frequency ω will be complex in the general case. We note that the requirements of one-dimensionality demand that $\mathbf{k} = k\mathbf{n}$, where $\mathbf{n}$ is a real unit vector, and k is a complex number. Substitution of the exponential equations (1.5) in system (1.4) leads to a homogeneous algebraic system

$$\begin{gathered}\mathbf{n}\times\mathbf{E}' = i\varkappa^{-1}\xi\mathbf{H}' \\ \mathbf{n}\times\mathbf{H}' = -i\varkappa^{-1}[(1+\xi)\mathbf{E}' + (\lambda-1)(\mathbf{l}\mathbf{E}')\mathbf{l}] \\ (\xi = -i\omega/4\pi\sigma,\ \varkappa = ck/4\pi\sigma,\ \lambda = \sigma_d/\sigma = d\ln j/d\ln E)\end{gathered} \tag{1.6}$$

The dispersion equation corresponding to Eq. (1.6) has the form

$$\begin{gathered}\xi P_2(\xi)\, P_3(\xi) = 0 \\ P_2(\xi) = \xi^2 + \xi + \varkappa^2,\quad P_3(\xi) = \xi^3 + (1+\lambda)\,\xi^2 + (\lambda + \varkappa^2)\,\xi + \\ + \varkappa^2(\sin^2\alpha + \lambda\cos^2\alpha)\end{gathered} \tag{1.7}$$

Here, α is the angle between vectors $\mathbf{E}_0$ and $\mathbf{n}$. For $\lambda \neq 1$ the roots of the polynomial $P_3(\xi)$ will depend on the angle α. Therefore, for difference in true and differential conductivity together with an anisotropic connection between the vectors $\delta\mathbf{j}$ and $\delta\mathbf{E}$, anisotropy occurs in the propagation of small disturbances.

We note that the root $\xi = 0$ in Eq. (1.7) must be discarded, since it corresponds to the nontrivial solution $\mathbf{E}' = 0$, $\mathbf{H}' = H'\mathbf{n} \neq 0$, incompatible at $\varkappa \neq 0$ with the requirement div $\delta\mathbf{H} = 0$. Those disturbances which are of interest are determined by the roots of the polynomials $P_2(\xi)$ and $P_3(\xi)$.

We will first examine the question of stability of the state of Eq. (1.3), examining the complex function $\omega(\mathbf{k})$ for real values of the argument. The function $\omega(\mathbf{k})$ is multivalued, and each of its single-valued branches $\omega_p(\mathbf{k})$ corresponds to one of the roots ξ_p of Eq. (17). The stability requirement consists of attaining for all real $\mathbf{k}$ the condition

$$\mathrm{Im}\,\omega_p(\mathbf{k}) < 0 \qquad (p = 1, 2, \ldots, 5) \tag{1.8}$$

The stationary state will be unstable if for some real $\mathbf{k}_0$ even one of the values $\omega_p(\mathbf{k}_0)$ falls in the upper semispace Im $\omega > 0$.

In as much as $\xi = -i\omega/4\pi\sigma$ and $\sigma > 0$, the stability requirements are equivalent to Re $\xi_p < 0$ for any real $\varkappa = ck/4\pi\sigma$, $0 \leq \alpha \leq \pi$.

The roots of the polynomial $P_2(\xi)$ have the form

$$\xi_{1,2} = -{}^1\!/_2\,(1 \pm \sqrt{1-4\varkappa^2}) \tag{1.9}$$

From Eq. (1.9) for all real $\varkappa$ it follows that:

$$\mathrm{Re}\,\xi_1 < 0, \qquad \mathrm{Re}\,\xi_2 \leqslant 0$$

For $\varkappa \neq 0$ transverse waves correspond to these roots, in which $\delta\mathbf{E} \parallel (\mathbf{n}\times\mathbf{l})$, $\delta\mathbf{H} \parallel (\mathbf{n}\times\delta\mathbf{E})$. The parallel orientation of the vector $\delta\mathbf{E}$ to a defined direction distinguished these disturbances from transverse waves in a medium with linear ohmic law, where $\delta\mathbf{E}$ can have any direction with respect to the plane of the wave front. In a nonlinear conductor with $\lambda \neq 1$, elliptical polarization of disturbances is possible only for the directions $\mathbf{n} = \pm\mathbf{l}$. Some similarity can be observed in the behavior of electromagnetic waves in uniaxial crystals.

For $\varkappa = 0$ to the root ξ_1 there corresponds a solution with $\delta\mathbf{H} = 0$, and a homogeneous disturbance $\delta\mathbf{E}$, damping over the classic time $1/4\pi\sigma$. The root ξ_2 for $\varkappa = 0$ tends to zero, and to it corresponds the solution $\delta\mathbf{E} = 0$, $\delta\mathbf{H} = \text{const}$. Thus, the disturbances related to the roots of the polynomial $P_2(\xi)$ do not lead to instability.

We note that the values of the roots $\xi_{1,2}$ themselves are obtained just as in the case of a linearly conductive medium, in which case every root is twofold.

The new (in comparison with classic) branches of the function $\omega(\mathbf{k})$ can correspond to the roots of the polynomial $P_3(\xi)$. Inasmuch as this polynomial is invariant for a replacement of α by $\pi - \alpha$, it is sufficient to examine the values $0 \leq \alpha \leq \frac{1}{2}\pi$. For arbitrary values of α the analytic expressions for the roots are difficult to examine. Therefore, we will present the solution for the two limiting cases ($\alpha = 0$ and $\alpha = \frac{1}{2}\pi$), and for intermediate angle values we will limit ourselves to a careful qualitative analysis.

For the case $\alpha = 0$, the roots of $P_3(\xi)$ have the form

$$\xi_3 = \xi_1, \quad \xi_4 = \xi_2, \quad \xi_5 = -\lambda$$

where ξ_1 and ξ_2 are determined by Eq. (1.9). The binary roots ξ_3 and ξ_4 correspond to damped transverse waves with elliptical or arbitrary linear polarization. The root ξ_5 corresponds to a standing longitudinal wave: $\delta\mathbf{E} \parallel \mathbf{n}$, $\delta\mathbf{H} = 0$, growing with time for $\lambda < 0$ and damping for $\lambda > 0$.

For $\alpha = \frac{1}{2}\pi$, we obtain the following expression for the roots:

$$\xi_{3,4} = -\tfrac{1}{2}\lambda\left(1 \pm \sqrt{1 - 4\varkappa^2/\lambda^2}\right), \quad \xi_5 = -1$$

The roots $\xi_{3,4}$ for $\varkappa \neq 0$ correspond to linearly polarized transverse waves ($\delta\mathbf{E} \parallel \mathbf{l}$, $\delta\mathbf{H} \parallel (\mathbf{n} \times \mathbf{l})$), increasing for $\lambda < 0$, and damping for $\lambda > 0$. The root ξ_5 corresponds to a standing longitudinal wave ($\delta\mathbf{E} \parallel \mathbf{n}$, $\delta\mathbf{H} = 0$) damping over the time $1/4\pi\sigma$.

Thus, a conclusion on the instability of the state of Eq. (1.3) for $\sigma_d < 0$ follows from the limiting cases examined. The increasing disturbances $\delta\mathbf{E}$ may be either potential, or turbulent. It is characteristic that in the examples above, of increasing waves, the vector $\delta\mathbf{E}$ is parallel to the stationary field $\mathbf{E}_0$. The presence of increasing standing waves in the case $\sigma_d < 0$ permits the conclusion that the instability is absolute (in the terminology of [10]). An analogous derivation may be arrived at on the basis of general criteria examined in [11].

Now it must be clarified whether the condition $\sigma_d > 0$ guarantees the fulfillment of the inequalities of Eq. (1.8) for arbitrary real $\mathbf{k}$. Furthermore, the determination of the region of wave vector values in which fluctuations will damp, even for negative σ_d, is of interest.

No matter what the sign of σ_d, disturbances related to the roots of polynomial $P_2(\xi)$ will not increase. Therefore, for an answer to these questions one must find the necessary and sufficient conditions under which all roots of the polynomial $P_3(\xi)$ lie in the left semispace $\operatorname{Re}\xi < 0$. Therefore, it is convenient to employ the Routh–Hurwitz stability criteria [12], which lead to the following inequalities:

$$1 + \lambda > 0, \quad \varkappa^2(\sin^2\alpha + \lambda\cos^2\alpha) > 0 \tag{1.10}$$

$$\lambda(1 + \lambda) + \varkappa^2(\cos^2\alpha + \lambda\sin^2\alpha) > 0$$

From Eq. (1.10) it follows that for $\lambda > 0$ and $\varkappa \neq 0$ any disturbances of the type of Eq. (1.5) damp over time. For $\lambda > 0$, $\varkappa = 0$, one of the roots of the polynomial $P_3(\xi)$ is equal to zero, while the two others are negative. To the zero root there corresponds a "neutral" disturbance $\delta\mathbf{H} = \text{const}$, $\delta\mathbf{E} = 0$. Thus, the state of Eq. (1.3) for $\sigma_d > 0$ is stable in relation to any sinusoidal disturbance.

For $\sigma_d < 0$ a region of wave vector values $\mathbf{k}$, wherein all disturbances will be damped, exists if the following conditions are fulfilled:

$$|\lambda| < \min(1, \varkappa^2), \quad \alpha_- < \alpha < \alpha_+ \quad (\pi - \alpha_+ < \alpha < \pi - \alpha_-) \tag{1.11}$$

$$\alpha_- = \operatorname{arc\,tg}(|\lambda|^{1/2}), \quad \alpha_+ = \operatorname{arc\,tg}\{(\varkappa^2 - |\lambda| + \lambda^2)^{1/2}|\lambda|^{-1/2}(1 - |\lambda| + \varkappa^2)^{-1/2}\}$$

From Eq. (1.11) we conclude that for $|\sigma_d| < \sigma$ for sufficiently short waves ($k^2 > k_0{}^2 = 16\pi^2\sigma|\sigma_d|$) there exists a region of directions of wave vectors M, in which all disturbances damp out with time. The region M is contained between two circular cones with aperture angles α_+ and α_-, and a common axis directed along the stationary field $\mathbf{E}_0$. For $\lambda \to 0$, $\alpha_- \to 0$, $\alpha_+ \to \frac{1}{2}\pi$, so that only long waves ($k^2 < k_0{}^2$) can increase, dependent on direction close to the common axis of the cones and close to the plane orthogonal to this axis. For an increase in $|\lambda|$ the field of stable directions narrows, disappearing at $\lambda = -1$, when merger of the two boundaries of the area M with the conic surface $\alpha = \frac{1}{4}\pi$ occurs.

We note the following peculiarity of the nontrivial solutions of system (1.6), corresponding to roots of the polynomial $P_3(\xi)$ for $\sin 2\alpha \neq 0$. In waves described by these solutions, $\mathbf{n}\delta\mathbf{E} \neq 0$ and $\mathbf{n} \times \delta\mathbf{E} \neq 0$. Consequently, in relation to disturbances of the electric field, such waves will be neither strictly longitudinal nor strictly transverse. (An exception is the case possible for $\lambda < 0$, when $\mathrm{tg}^2\alpha = -\lambda$. In this case, one of the roots of the polynomial $P_3(\xi)$ goes to zero, and there is a corresponding disturbance $\delta\mathbf{E} \parallel \mathbf{n}$, $\delta\mathbf{H} \parallel (\mathbf{n} \times \mathbf{l})$.)

When displacement currents are not considered, Eq. (1.4) becomes nonevolutionary for the case $\sigma_d < 0$. A corresponding example was developed in [4], where the possibility was also indicated of limiting the growth rate of shortwave disturbances due to displacement currents. The requirement of evolution implies that all the functions $\omega_p(\mathbf{k})$ must, for $k \to \infty$, satisfy the inequalities

$$\mathrm{Im}\,\omega_p(\mathbf{k}) < \mathrm{const} \tag{1.12}$$

An evaluation of Im $\omega_p(\mathbf{k})$ for the case of arbitrarily oriented disturbances was not performed in [4]. Such evaluation can be obtained by examination of the dispersion equation Eq. (1.7). The functions $\omega_{1,2}(\mathbf{k})$ corresponding to the roots $\xi_{1,2}$ are known to satisfy the requirements of Eq. (1.12), since they always possess a nonpositive imaginary term. For the functions $\omega_p(\mathbf{k})$ corresponding to the roots of the polynomial $P_3(\xi)$, the following asympotic behavior occurs for $k \to \infty$:

$$\omega_{3,4}(\mathbf{k}) = \pm ck - 2\pi\sigma(\cos^2\alpha + \lambda\sin^2\alpha)\,i + O(k^{-1})$$
$$\omega_5(\mathbf{k}) = -4\pi\sigma(\sin^2\alpha + \lambda\cos^2\alpha)\,i + O(k^{-1})$$

Therefore, inequality (1.12) is satisfied for all branches of the functions $\omega(\mathbf{k})$, which permits making a conclusion on the evolubility of system (1.4), irrespective of the sign of the differential conductivity.

Closely connected to the stability problem examined above is the question of the possibility of amplifying oscillations imposed on the stationary state of Eq. (1.3). The physical basis of the amplification problem consists of clarifying the conditions under which waves propagating from an oscillatory source will grow in space with distance traversed. To find such conditions, it is necessary to examine all branches k_p of the multivalued function $k(\omega, \alpha)$ for real ω. The damping condition for harmonic oscillations in space consists of fulfilling, for all real $\omega \neq 0$ and $0 \leq \alpha \leq \frac{1}{2}\pi$, the inequalities

$$\omega \mathrm{Re} k_p \mathrm{Im} k_p > 0 \tag{1.13}$$

for all functions $k_p(\omega, \alpha)$. If Eq. (1.13) is satisfied, the amplitude of traveling waves damps in the direction of the phase velocity $\mathbf{v} = (\omega/\mathrm{Re}\, k)\,\mathbf{n}$.

For plane waves of the type of Eq. (1.5) with a real value of ω, we will examine the energy flux vector, averaged over the period of the oscillation $T = 2\pi/\omega$

$$\mathbf{s} = T^{-1}\int_t^{t+T} \frac{c}{4\pi}(\delta\mathbf{E} \times \delta\mathbf{H})\,dt = \frac{c}{16\pi}(\mathbf{E}' \times \mathbf{H}_*{}' + \mathbf{E}_*{}' \times \mathbf{H}') \tag{1.14}$$

The asterisk denotes the operation of complex conjugation. The physical basis of the requirements of Eq. (1.13) is that wave amplitude, damping in the direction $\mathbf{V}$, be equivalent to the damping in the direction of energy transfer, i.e., under the condition that $\mathbf{vs} > 0$. The latter condition is actually fulfilled, since from Eq. (1.14) and the first equation of (1.6) it follows that

$$\mathbf{s} = (16\pi\omega)^{-1}c^2\left[(k + k_*)(\mathbf{E}'\mathbf{E}_*{}')\,\mathbf{n} - k_*(\mathbf{E}'\mathbf{n})\,\mathbf{E}_*{}' - k(\mathbf{E}_*{}'\mathbf{n})\,\mathbf{E}'\right]$$
$$\mathbf{vs} = (8\pi)^{-1}c^2(|\mathbf{E}'|^2 - |\mathbf{E}'\mathbf{n}|^2) \geqslant 0$$

For linear electromagnetic waves in an anisotropic medium the condition $\mathbf{vs} \geq 0$ was derived in [13]. In essence, the derivation of this result rests only on the first equation of (1.4) and the assumption of parallelism between the vectors Re k and Im k. In a nonlinear conductor, waves of small amplitude possessing a phase velocity cannot be purely longitudinal, so that the value vs proves to be strictly positive.

Using the identity

$$\mathrm{Re}\, k\, \mathrm{Im}\, k = {}^1/_2\, \mathrm{Im}\, k^2$$

we transform inequality (1.13) to the more convenient form

$$\Omega\, \mathrm{Im}\, \varkappa_p^2 > 0 \tag{1.15}$$

$$(\Omega = \omega/4\pi\sigma,\ \varkappa_p^2 = c^2k_p^2/16\pi^2\sigma^2)$$

From the dispersion equation (1.7), two possible values are determined for $\varkappa^2$: $\varkappa_1^2$ from the condition $P_2(\xi;\ \varkappa^2) = 0$, $\varkappa_2^2$ from the condition $P_3(\xi;\ \varkappa^2,\ \alpha) = 0$, where $\xi = -i\Omega$. For the value $\Omega\varkappa_1^2$, we obtain the expression

$$\Omega\varkappa_1^2 = \Omega^3 + i\Omega^2$$

satisfying inequality (1.15). The value of $\Omega\varkappa_2^2$ is determined by the formula

$$\Omega\varkappa_2^2 = [\Omega^2 + (\sin^2\alpha + \lambda\cos^2\alpha)^2]^{-1}\ \{\Omega^5 + (\sin^2\alpha + \lambda^2\cos^2\alpha)\,\Omega^3 + \\ + i\,[(\cos^2\alpha + \lambda\sin^2\alpha)\,\Omega^4 + \lambda\,(\sin^2\alpha + \lambda\cos^2\alpha)\,\Omega^2]\}$$

From the last expression it follows that condition (1.15) is always satisfied for $\lambda > 0$. Therefore, the requirement $\sigma_d > 0$ is a sufficient condition for absorption of waves in the medium. On the other hand, for $\sigma_d < 0$ and $\alpha = {}^1\!/_2\pi$ the requirement of (1.15) is broken, and the amplitude of oscillations will increase along the path of the waves.

Thus, the requirement $\sigma_d > 0$ is simultaneously the criterion for stability of the homogeneous state, and the criterion for absorption of waves propagating from a source of periodic oscillations. Furthermore, the range of directions n, in which absorption of oscillations of a given frequency occurs, differs for the case $\sigma_d < 0$ from the range of directions determined by Eq. (1.11), which corresponds to damping over time of periodic disturbances along n with a given wavelength. In reality, for $\lambda < 0$ inequality (1.15) is fulfilled for the following range of values of α:

$$0 \leqslant \alpha < \alpha_0, \quad \pi - \alpha_0 < \alpha \leqslant \pi \\ \alpha_0 = \mathrm{arc\ tg}\,[(\lambda^2 + \Omega^2)^{1/2}\,|\lambda|^{-1/2}(1 + \Omega^2)^{-1/2}] \tag{1.16}$$

A range of absorption directions exists for any $\sigma_d < 0$ and fills the interior of a cone, the axis of which is parallel to the vector $\mathbf{E}_0$, and whose aperture is equal to α_0. For fixed $\lambda \neq -1$ the angle α_0 will be a monotonic function of Ω^2, so that for an increase in Ω^2 from zero to ∞ the value of α_0 changes from $\alpha_0 = \alpha_* = \arctan(|\lambda|^{1/2})$ to $\alpha_0 = {}^1\!/_2\pi - \alpha_*$. With increase in frequency the field of directions for oscillation amplification contracts, if $|\lambda| < 1$, and expands for $|\lambda| > 1$. The lines of constant value for the quantity $\eta = \tan^2\alpha_0$ in the plane $(|\lambda|,\ \Omega^2)$ are depicted in Fig. 1.

2. The Problem of Stability in a Bounded Region. Energy Relationships

We will examine the stability of the state of Eq. (1.3) in a cylindrical region $D \equiv \{(x_1,\ x_2) \in G,\ |x_3| < h\}$, where x_i are Cartesian coordinates, and G is an arbitrary two-dimensional region.

The state of Eq. (1.3) is realized by passage of a constant current through the conductor from an outside source. We will consider the ends of the cylinder to be ideally conductive, while its lateral surface and the external network are surrounded by a nonconductive medium close to a vacuum in dielectric properties. In principle, the electric field outside the region D and the induced magnetic field in the entire space can be uniquely determined from the stationary field equations with the usual boundary conditions at the interface of the media and at infinity.

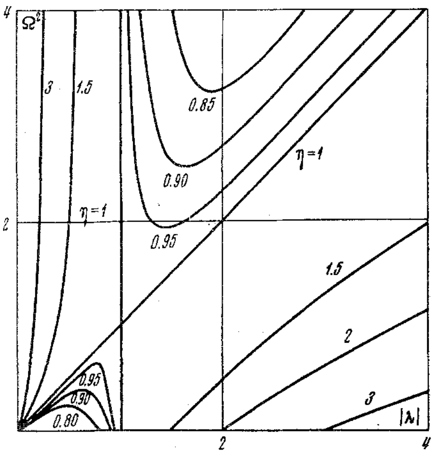

Fig. 1

In studying the problem of stationary-state stability it is natural to examine fluctuations which meet the following requirements:

1) the initial conditions together with the physical conditions at the boundaries of the medium uniquely determine the disturbance field throughout the space for $t > 0$;

2) the disturbances examined cannot arise in the region D for $t > 0$, if they are absent at $t = 0$.

Disturbances satisfying these conditions will be called admissible. In order to ensure the satisfaction of requirements 1) and 2), we will examine the Cauchy problem for disturbances throughout the entire space, assuming that the initial distributions

$$\delta \mathbf{E}(\mathbf{x}, 0) = \delta \mathbf{E}_0(\mathbf{x}), \qquad \delta \mathbf{H}(\mathbf{x}, 0) = \delta \mathbf{H}_0(\mathbf{x}) \tag{2.1}$$

are sufficiently smooth, and identically equal to zero outside region D, and at the boundary S.

The field disturbances $\delta \mathbf{E}^-$, $\delta \dot{\mathbf{H}}^-$ within D and $\delta \mathbf{E}^+$, $\delta \mathbf{H}^+$ outside D must satisfy the condition of continuity of tangential components on the boundary of the region

$$\delta \mathbf{E}_\tau^- = \delta \mathbf{E}_\tau^+ = 0 \qquad (\mathbf{x} \in S_1)$$
$$\delta \mathbf{E}_\tau^- = \delta \mathbf{E}_\tau^+, \qquad \delta \mathbf{H}_\tau^- = \delta \mathbf{H}_\tau^+ \qquad (\mathbf{x} \in S_2) \tag{2.2}$$

Here, S_1 are the face electrodes; S_2, the lateral surface of the cylinder.

With such a formulation, disturbances concentrated within the region D for $t = 0$ can propagate into the surrounding medium for $t > 0$. The disturbance field will be equal to zero within the changing region D_*, bounded by the forward wave front S_*, propagating at the speed of light. The closed surface S_* is a characteristic, and at a moment in time t is an envelope of a family of spherical fronts of radius ct with centers at points in the boundary of S. As a consequence of the continuity of the initial distributions of Eq. (2.1), equal to zero on S, the disturbance field at the wave front S_* will tend to zero

$$\delta \mathbf{E}(\mathbf{x}_*, t) = \delta \mathbf{H}(\mathbf{x}_*, t) = 0 \qquad (\mathbf{x}_* \in S_*) \tag{2.3}$$

For small disturbances, a result may be obtained analogous to the Poynting theorem of conventional electrodynamics [14]

$$\int_V \frac{\partial w}{\partial t} dV = -\int_V \delta \mathbf{j} \delta \mathbf{E} dV - \frac{c}{4\pi} \int_\Sigma (\delta \mathbf{E} \times \delta \mathbf{H}) \, \mathbf{v} d\Sigma$$
$$w = [(\delta \mathbf{E})^2 + (\delta \mathbf{H})^2] / 8\pi \tag{2.4}$$

Here, V is an arbitrary volume, bounded by the surface Σ, and ν is the external normal to Σ. In the derivation of Eq. (2.4) the first two equations of system (1.4) and the continuity condition (2.2) were used. Let V be the volume of the complete disturbance field D_*. Then, the reduction to zero of the surface integral in Eq. (2.4) follows from Eq. (2.3). Neglecting dissipation of the disturbance in the external network due to the relatively high conductivity of its elements, we limit the volume integral in the right side of Eq. (2.4) to the region D. As a result, we have

$$\int_{D_*} \frac{\partial w}{\partial t} dD = -\int_D \delta \mathbf{j} \delta \mathbf{E} dD$$

Using the formula for time differentiation of the integrals taken over the moving volume D_*

$$\frac{d}{dt} \int_{D_*} f dD = \int_{D_*} \frac{\partial f}{\partial t} dD + c \int_{S_*} f dS \tag{2.5}$$

assuming herein that $f = w$ and considering Eq. (2.3), we proceed to the equation

$$\frac{dW_*}{dt} = -\int_D [\sigma_d (\delta E_\parallel)^2 + \sigma (\delta E_\perp)^2]\, dD \tag{2.6}$$

$$W_* = \int_{D_*} w dD, \quad \delta E_\parallel = (l \delta E)\, l, \quad \delta E_\perp = \delta E - \delta E_\parallel, \quad l = E_0 / E_0$$

As a consequence of the linearity of Eqs. (2.1), (2.2) for disturbances, the continuous solution is unique, if the equation with neutral initial conditions [Eq. (2.1)] has only a trivial solution. It follows from Eq. (2.6) that

$$dW_* / dt \leqslant 8\pi\sigma_m W_*, \quad \sigma_m = \max(\sigma, |\sigma_d|)$$

Integrating this inequality, we obtain $W_*(t) \leq W_*(0) \exp 8\pi\sigma_m t$. For $W_*(0) = 0$ the function $W_*(t)$ remains equal to zero at subsequent times, and consequently, the solution is unique. This result remains valid even for the case of an inhomogeneous stationary state, when the electrode location or geometry of the region D do not permit a solution of Eq. (1.3). For an inhomogeneous current distribution, instead of the constant σ_m, one must take max $\sigma_m(\mathbf{x})$ for $\mathbf{x} \in D$.

We will define the stationary state in region D as stable if, on the average, for $t \to \infty$ the energy of admissible disturbances in this region satisfies the condition

$$W(t) = \int_D w dD \leqslant W(0) \tag{2.7}$$

By instability of the stationary state in D we understand the ability of admissible disturbances to increase over time. For $\sigma_d < 0$ there always exist disturbances whose energy increases over the course of some initial time interval. In fact, examining disturbances for which $\delta E_\perp = 0$ at $t = 0$, from Eq. (2.6), we obtain $dW/dt > 0$ for $t = 0$.

For the case $\sigma_d < 0$ the existence of disturbances $\delta\mathbf{E}$ increasing for $t \to \infty$ in D is easy to establish for the homogeneous state of Eq. (1.3), since system (1.4) admits a solution on D of the form

$$\delta\mathbf{E} = \delta E_0(x_3) \exp(-4\pi\sigma_d t)\, \mathbf{e}_3, \qquad \delta\mathbf{H} \equiv 0$$

satisfying the condition $\delta\mathbf{E}_\tau$ at the electrodes. We note that for potential disturbances in D of $\delta\mathbf{E}$ the condition $\delta E_0(\mathbf{x}) = 0$ on the entire boundary S becomes unnecessary, otherwise the problem of determining the potential will become incorrect.

An example of turbulent disturbance increasing without limit is easily constructed for the case of a conductive layer between two infinite plane electrodes $|x_3| < h$. We will seek a turbulent solution for $\delta\mathbf{E}$ in the form

$$\delta\mathbf{E} = u(x_1, x_2)\, g(t)\, \mathbf{e}_3$$

The boundary condition $\delta\mathbf{E}_\tau = 0$ will now be fulfilled automatically. Moreover, in this case, the problem of inner disturbance is completely separate from that of outer. Eliminating from Eq. (1.4) the vector $\delta\mathbf{H}$, we proceed to the equations

$$\nabla^2 u - \mu u = 0, \qquad g'' + 4\pi\sigma_d g' - c^2\mu g = 0$$

A finite solution to the Helmholtz equation, damping at infinity, may be taken as

$$u = u(r) = J_0(\sqrt{-\mu}\, r), \qquad r = \sqrt{x_1^2 + x_2^2}, \qquad \mu < 0$$

The equation for the function g(t) has the following general solution:

$$g = C_1 \exp \zeta_1 t + C_2 \exp \zeta_2 t$$

$$\zeta_{1,2} = -2\pi\sigma_d \pm (4\pi^2\sigma_d^2 + c^2\mu)^{1/2}$$

Inasmuch as Re $\zeta_{1,2} > 0$ for $\sigma_d < 0$, the solutions found increase for $t \to \infty$.

From Eq. (2.6) it follows that for admissible disturbances $dW_*/dt \le 0$, if $\sigma_d > 0$. Therefore, $W_*(t) \le W_*(0) = W(0)$, and the condition of Eq. (2.7) will be filled in this case, due to the inequality $W(t) \le W_*(t)$. For stationary current distributions, not necessarily homogeneous, a more powerful result can be obtained—asymptotic stability of these distributions for $\sigma_d > 0$, i.e.,

$$\lim_{t\to\infty} W_E(t) = 0, \quad W_E(t) = \frac{1}{8\pi}\int_D (\delta \mathbf{E})^2 dD \tag{2.8}$$

If Eq. (2.8) is fulfilled, disturbances $\delta\mathbf{E}$ and $\delta\mathbf{j}$ tend to zero for $t\to\infty$ practically everywhere in the region D.

We will now turn to a proof of the assertion of Eq. (2.8). We assume that the field of disturbances $\delta\mathbf{E}$ and δH is continuously differentiable twice with respect to x and t within and without the region D. If at the ideally conductive portions of the boundary S there occurs a discontinuity of the component δE_ν or δH_τ, we will assume that this does not lead to violation of the usual rules of differentiation with respect to time of the field integrals, taken over a fixed volume. (In the majority of cases these rules are preserved even for spatially discontinuous distributions.) The function $W_*(t)$ in the case $\sigma_d > 0$ is nonincreasing and limited below ($W_* \ge 0$). Therefore, for $t\to\infty$, there exists a $\lim W_*(t) = W_*(\infty)$. Consequently, the improper integral

$$I = \int_0^\infty (dW_*/dt)\,dt = W_*(\infty) - W_*(0) \tag{2.9}$$

converges.

From Eq. (2.6) there follows the approximation

$$0 \leqslant W_E \leqslant (8\pi\sigma_*)^{-1}|dW_*/dt|, \qquad \sigma_* = \min_{x\in D}(\sigma, \sigma_d)$$

Therefore, instead of Eq. (2.8), it is sufficient to prove that

$$\lim_{t\to\infty} dW_*/dt = 0 \tag{2.10}$$

The condition of Eq. (2.10) will exist, if along with convergence of the integral I the derivative of the integrand

$$|d^2W_*/dt^2| < \text{const} \qquad (0 < t < \infty) \tag{2.11}$$

is uniformly limited.

We will establish the validity of Eq. (2.11). Differentiating Eq. (2.6) with respect to time, we obtain

$$\frac{d^2W_*}{dt^2} = -2\int_D \left(\sigma_d \delta\mathbf{E}_\parallel \frac{\partial}{\partial t}\delta\mathbf{E}_\parallel + \sigma\delta\mathbf{E}_\perp \frac{\partial}{\partial t}\delta\mathbf{E}_\perp\right) dD$$

We will evaluate the right side of the last inequality

$$\left|\frac{d^2W_*}{dt^2}\right| \leqslant 2\sigma^* \int_D |\delta\mathbf{E}|\left|\frac{\partial}{\partial t}\delta\mathbf{E}\right| dD \leqslant 2\sigma^*\left(\int_D (\delta\mathbf{E})^2 dD\right)^{1/2}\left(\int_D\left(\frac{\partial}{\partial t}\delta\mathbf{E}\right)^2 dD\right)^{1/2} \leqslant$$

$$\leqslant 2\sigma^* \sqrt{8\pi W(0)}\left(\int_D \left(\frac{\partial}{\partial t}\delta\mathbf{E}\right)^2 dD\right)^{1/2}, \qquad \sigma^* = \max_{x\in D}(\sigma, \sigma_d) \tag{2.12}$$

The Cauchy–Bunyakovski inequality was utilized here, as well as the condition $W_E(t) \le W_*(t) \le W_*(0) = W(0)$. It remains to be proved that the volume integral of $(\partial\delta E/\partial t)^2$ is uniformly limited over t. To achieve this, we eliminate the vector δH from system (1.4) by differentiation. As a result, we have the following equation:

$$\frac{\partial^2}{\partial t^2}\delta\mathbf{E} + c^2 \operatorname{rot}\operatorname{rot}\delta\mathbf{E} + 4\pi\left(\sigma_d\frac{\partial}{\partial t}\delta\mathbf{E}_\parallel + \sigma\frac{\partial}{\partial t}\delta\mathbf{E}_\perp\right) = 0 \tag{2.13}$$

Equation (2.13) examines the conductive region D and the portion of the vacuum occupied by the disturbance, where $\sigma = \sigma_d = 0$. Scalarly multiplying Eq. (2.13) by $\partial\delta E/\partial t$, and integrating the equality obtained over the region D_*, we have

$$\frac{1}{8\pi}\int_{D_*}\frac{\partial}{\partial t}\left[\left(\frac{\partial}{\partial t}\delta\mathbf{E}\right)^2 + c^2(\mathrm{rot}\,\delta\mathbf{E})^2\right]dD = -\int_D\left[\sigma_d\left(\frac{\partial}{\partial t}\delta\mathbf{E}_\parallel\right)^2 + \sigma\left(\frac{\partial}{\partial t}\delta\mathbf{E}_\perp\right)^2\right]dD - \frac{c}{4\pi}\int_{S_*}\left(\frac{\partial}{\partial t}\delta\mathbf{E}\times\frac{\partial}{\partial t}\delta\mathbf{H}\right)\mathbf{v}\,dS \tag{2.14}$$

If the conditions at the wave front [Eq. (2.3)] are differentiated with respect to time and a result of the Maxwell equations in a vacuum for a point on the surface S_* is used, we obtain

$$\left(\frac{\partial}{\partial t}\delta\mathbf{E}\times\frac{\partial}{\partial t}\delta\mathbf{H}\right)\mathbf{v} = \frac{1}{2}\left[\left(\frac{\partial}{\partial t}\delta\mathbf{E}\right)^2 + \left(\frac{\partial}{\partial t}\delta\mathbf{H}\right)^2\right], \qquad \mathbf{x}\in S_* \tag{2.15}$$

Using Eq. (2.15) in Eq. (2.5), in which we assume

$$f = \frac{1}{8\pi}\left[\left(\frac{\partial}{\partial t}\delta\mathbf{E}\right)^2 + c^2(\mathrm{rot}\,\delta\mathbf{E})^2\right] = \frac{1}{8\pi}\left[\left(\frac{\partial}{\partial t}\delta\mathbf{E}\right)^2 + \left(\frac{\partial}{\partial t}\delta\mathbf{H}\right)^2\right]$$

Eq. (2.14) takes on the form

$$\frac{dR}{dt} = -\int_D\left[\sigma_d\left(\frac{\partial}{\partial t}\delta\mathbf{E}_\parallel\right)^2 + \sigma\left(\frac{\partial}{\partial t}\delta\mathbf{E}_\perp\right)^2\right]dD$$

$$R(t) = \frac{1}{8\pi}\int_{D_*}\left[\left(\frac{\partial}{\partial t}\delta\mathbf{E}\right)^2 + \left(\frac{\partial}{\partial t}\delta\mathbf{H}\right)^2\right]dD$$

From the last equation it follows that $R(t) \le R(0)$ for $t > 0$. Then, from Eq. (2.12), we obtain the evaluation required

$$|d^2W_*/dt^2| \leqslant 16\pi\sigma^*\sqrt{W(0)R(0)}$$

The constants W(0) and R(0) are determined by setting initial distributions of Eq. (2.1).

We will now examine the behavior of the integral Joulean dissipation in the region D for application of small disturbances on the homogeneous state of Eq. (1.3). First, we will clarify the conditions under which the stationary state is characterized by a minimum in integral dissipation in comparison to the nonstationary state produced by application of the initial disturbances. In order that the function

$$Q = \int_D \mathbf{E}\mathbf{j}(\mathbf{E})\,dD \equiv \int_D q(E)\,dD$$

examined in some class of fields, have a minimum for the stationary solution $\mathbf{E}_0$, its first variation must go to zero

$$\delta Q = \int_D(\sigma + \sigma_d)\,\mathbf{E}_0\delta\mathbf{E}\,dD = 0$$

From Eq. (2.13), it is possible to obtain the relationship

$$\frac{d^2}{dt^2}\delta Q + 4\pi\sigma_d\frac{d}{dt}\delta Q = c^2(\sigma + \sigma_d)\int_S(\mathbf{E}_0\times\mathrm{rot}\,\delta\mathbf{E})\,\mathbf{v}\,dS$$

which indicates that the requirement $\delta Q = 0$ cannot be fulfilled, if arbitrary disturbances admissible in the analysis of stability are considered. The condition $\delta Q = 0$ is fulfilled for potential disturbances $\delta\mathbf{E} = -\nabla\delta\varphi$ in the case when the stationary field $\mathbf{E}_0$ is homogeneous, and a constant potential difference is maintained on the electrodes. In fact, under these conditions

$$\delta Q = -(\sigma + \sigma_d)\int_D \mathrm{div}\,(\mathbf{E}_0\delta\varphi)\,dD = (\sigma + \sigma_d)\,E_0 G\delta(\varphi_+ - \varphi_-) = 0$$

Here, G is the electrode area, and $\delta(\varphi_+ - \varphi_-)$ is the interelectrode potential disturbance. If $\delta Q = 0$, then minimum Q will be realized in the stationary solution of Eq. (1.3) under the condition that

$$\delta^2 Q = \frac{1}{2}\int_D \left(\frac{\partial^2 q}{\partial E_i \partial E_j}\right)\delta E_i \delta E_j dD = \frac{1}{2}\int_D \left[\frac{d^2 q}{dE^2}(\delta \mathbf{E}_\parallel)^2 + \frac{1}{E}\frac{dq}{dE}(\delta \mathbf{E}_\perp)^2\right] dD > 0$$

The arbitrary constants appearing here are calculated for $\mathbf{E} = \mathbf{E}_0$. The condition $\delta^2 Q > 0$ is fulfilled for arbitrary disturbances, if at the point $E = E_0$

$$d^2 q / dE^2 = E d^2 j / dE^2 + 2 dj / dE > 0, \quad dq / dE = E dj / dE + j > 0 \tag{2.16}$$

In order to use the condition $\delta^2 Q > 0$ for any stationary solution, Eq. (2.16) must be fulfilled at all points of the volt–ampere characteristic. Then, the local dissipation q(E) will be a monotonically increasing convex function.

For potential disturbances $\delta \mathbf{E} = -\nabla \delta\varphi$, $\delta H = 0$ in a cylindrical region with face electrodes, of necessity $\delta \mathbf{E}_\perp = 0$, so that there is no need to examine the second inequality of Eq. (2.16).

The satisfaction of the first condition of Eq. (2.16) at all points of the volt–ampere curve is a stricter limitation on the shape of the function j(E), than the stability requirement $\sigma_d(E) > 0$.

We will show that for a single-valued function j(E), due to the positive sign of d^2q/dE^2, the function $\sigma_d(E)$ will be positive. We will assume that j(E) is a twice continuously differentiable function, and $\sigma(0) > 0$. We assume that the first inequality of Eq. (2.16) is satisfied everywhere, but there exists a value $E = E_1$, for which $\sigma_d(E_1) \leq 0$. Inasmuch as $\sigma_d(0) = \sigma(0) > 0$, then, $\sigma_d(E) > 0$ on some interval $0 < E < E_\delta$. The set of E_δ values is bounded above, since $\sigma_d(E_1) \leq 0$. Hence, it has an exact upper limit, E_2. From the continuity of the function $\sigma_d(E)$, and the properties of an upper limit, it follows that $\sigma_d(E_2) = 0$. For any E in the interval $(0, E_2)$, we have

$$-\sigma_d(E) = \sigma_d(E_2) = \sigma_d(E) - \left(\frac{d\sigma_d}{dE}\right)_{E=E_*}(E_2 - E), \quad E_* \in (E, E_2)$$

Thence, we conclude that in an arbitrarily small left-hand neighborhood of the point E_2, there are points E_*, at which $d\sigma_d/dE = d^2j/dE^2 < 0$. Then, it follows from the continuity of the function d^2j/dE^2 at the point E_2:

$$(d^2 j / dE^2)_{E=E_2} \leqslant 0$$

But at $E = E_2$, according to our assumption, the first inequality of Eq. (2.16) is fulfilled, which in view of the condition $\sigma_d(E_2) = 0$ takes on the form

$$(d^2 j / dE^2)_{E=E_2} > 0$$

The contradiction obtained demonstrates the impossibility of the existence of a point E_1.

The converse of the proposition proved above is not valid. It is simple to construct examples of the function j(E), for which $\sigma_d > 0$, but q(E) will not be a convex function at all points.

In [9] it was attempted to establish equivalency between the stability requirement $\sigma_d > 0$, and the principle of minimal entropy production in the stationary state. The latter condition, however, was replaced by the principle of minimum Joulean dissipation for a single-valued function j(E) and constant voltage on the electrodes. The application of special kinetic models shows that the stationary state is not characterized by minimum entropy production [2].

Within the framework of the model of a nonlinear conductive medium studied herein, the stability requirement $\sigma_d > 0$ follows from the requirement of minimum Q in any stationary state of Eq. (1.3), in the class of disturbances which reduce the first variation δQ to zero. However, this extremal principle, to speak in general, is not applicable to media with arbitrary positive differential conductivity. In connection with this, we note that the conclusion of coincidence in sign of the quantities $\delta^2 Q$ and σ_d, made in [9], is in error: in calculating the increment ΔQ for the case $\delta \mathbf{E} = \delta \mathbf{E}_\parallel$ a term of the form $\frac{1}{2}E(d^2j/dE^2)(\delta \mathbf{E}_\parallel)^2$ was lost, related to the curvature of the volt–ampere characteristic. The integral contribution of this term to $\delta^2 Q$ completely balances the second variation of the energy flux in the case of a nonlinear medium and does not effect the energy change rate of the disturbances.

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